

Optimization of Pool and Tournament Play in the Top Swedish Handball League

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Contributions

- Optimal schedule design for *Elitserien* to be used for 2013-14 season
- Necessary conditions for schedulability of HAP sets

Problem

- 14-team league; owners want more than 26 games, but not 39 games in a triple round robin
- Would like to form two divisions which hold internal single round-robin tournaments before the main tournament
- Want very fair home-away patterns
- Desire a template they can use on their own
- Otherwise standard requirements, so hopefully the results are generally useful

Home/Away Pattern Sets

- Tournament scheduling often simplified by constructing home/away pattern (HAP) sets
- Desirable home-away patterns for each team: number of breaks and when they occur

| | |
|--------|-------|
| Team 1 | AHAHA |
| Team 2 | AAHAH |
| Team 3 | AHHAH |
| Team 4 | HAHAH |
| Team 5 | HHAHA |
| Team 6 | HAAHA |

League Requirements

- Each 7-team division must hold a SRRT to start the season.
- This must be followed by two SRRTs between the entire league, the second being a mirror of the first.
- There must be a minimum number of breaks in the schedule.
- Each team has one bye during the season (to occur during the divisional RRT).
- The number of home and away games played by a team cannot differ by more than 1.
- Any pair of teams must have consecutive meetings occur at different venues. (AVR)
- Each division must have 3 pairs of complementary schedules.

An Optimal HAP Set

- Every n -team RRT, n even, must have at least $n - 2$ breaks, DeWerra (1981)
- For an n -team RRT, n odd, there exists a unique no break tournament, Fronček (2005)

| | |
|---------|----------------|
| BAHAHAH | AHAHAHAHAHAHA |
| HBAHAHA | AHAHAHAHAHAHH |
| AHBAHAH | AHAHAHAHAHHAH |
| HAHBAHA | AHAHAHAHHAAHA |
| AHAHBAB | AHAHAHHAHAHAH |
| HAHAHBA | AHAHHAHAHAHAH |
| AHAHAHB | AHAHHAHAHAHAH |
| or | AHHAHAHAHAHAH |
| BHAHAHA | HAAHAHAHAHAHA |
| ABHAHAH | HAHAHAHAHAHAHA |
| HABHAHA | HAHAHAHAHAHAHA |
| AHABHAH | HAHAHAHAHAHAHA |
| HAHABHA | HAHAHAHAHAHAHA |
| AHAHABH | HAHAHAHAHAHAHA |
| HAHAHAB | HAHAHAHAHAHAHA |

Not every HAP set is schedulable

Teams to Numbers

Additional requests and concerns can be addressed when assigning teams to numbers:

- Venue availabilities; desired derby games
- More meetings between the top teams and between the bottom teams in the last weeks.

This can be accomplished by solving a small integer program

Simple Condition

For a HAP set S to be schedulable, for any two teams t_1 and t_2 in the same division, there must be periods p_1 in Part I and p_2 in Part II such that

$$\begin{aligned} S(t_1, p_1) &= H & \text{and} & & S(t_2, p_1) &= A, \\ S(t_1, p_2) &= A & \text{and} & & S(t_2, p_2) &= H. \end{aligned}$$

Theorem

For an n -team tournament, $\frac{n}{2}$ odd, with a divisional RRT before a full-league DRRT, there are

$$\frac{n}{2} P_{\frac{n-2}{4}} \times \left(\frac{n+2}{4} \right)^3 \times \frac{n-2}{4}!$$

unique HAP sets satisfying the requirements, except possibly for the AVR, with $\frac{n-2}{4}$ pairs of complementary schedules within each division.

For the 14-team Elitserien: 80640 HAP sets.

Latin Square Condition

Build an $n \times n$ array, where each entry (i, j) is a vector of periods when it is possible for teams i and j to meet.

- If (i, j) has only one entry, remove that value (if possible) from any vector (i, k) , $k \neq j$ and any vector (k, j) , $k \neq i$.
- If (i, j) has more than one entry, see if any value is unique in a row or column. Replace (i, j) by that value.
- Stop if any (i, j) is empty, or no change is observed after checking the above two conditions for all (i, j) .

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|-------------|-------------|-------------|-----------|-----|---|
| 1 | | | | | | |
| 2 | [2,3,4,5] | | | | | |
| 3 | [3,4,5] | [2] | | | | |
| 4 | [1,2,3,4,5] | [1] | [1,2] | | | |
| 5 | [1] | [1,2,3,4,5] | [1,3,4,5] | [2,3,4,5] | | |
| 6 | [1,2] | [1,3,4,5] | [1,2,3,4,5] | [3,4,5] | [2] | |

Efficiency

| n | HAP Sets | Unschedulable | Simple Condition | Latin Square Condition |
|----|----------|---------------|------------------|------------------------|
| 6 | 24 | 20 | 8 (40%) | 20 (100%) |
| 10 | 1080 | 998 | 396 (40%) | 998 (100%) |
| 14 | 80640 | 79024 | 30720 (39%) | 75995 (96%) |

Result: Optimal Template

| | | | | | | | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|-----|-----|-----|----|-----|----|-----|-----|-----|-----|-----|----|-----|-----|
| 0 | -2 | 3 | -4 | 5 | -6 | 7 | -8 | 9 | -5 | 6 | -7 | 10 | -11 | 12 | -13 | 4 | -14 | 2 | -3 | ... |
| -7 | 1 | 0 | -3 | 4 | -5 | 6 | -9 | 7 | -12 | 10 | -6 | 5 | -4 | 11 | -14 | 13 | 3 | -1 | 8 | ... |
| -6 | 7 | -1 | 2 | 0 | -4 | 5 | -10 | 8 | -7 | 9 | -5 | 11 | -13 | 6 | -12 | 14 | -2 | 4 | 1 | ... |
| -5 | 6 | -7 | 1 | -2 | 3 | 0 | -11 | 10 | -6 | 5 | -13 | 12 | 2 | -14 | 8 | -1 | 9 | -3 | 7 | ... |
| 4 | 0 | -6 | 7 | -1 | 2 | -3 | 13 | -14 | 1 | -4 | 3 | -2 | 8 | -7 | 9 | -10 | -11 | 6 | -12 | ... |
| 3 | -4 | 5 | 0 | -7 | 1 | -2 | 12 | -13 | 4 | -1 | 2 | -8 | 14 | -3 | 7 | -9 | 10 | -5 | -11 | ... |
| 2 | -3 | 4 | -5 | 6 | 0 | -1 | 14 | -2 | 3 | -8 | 1 | -9 | -10 | 5 | -6 | 11 | -12 | 13 | -4 | ... |

| | | | | | | | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|----|-----|-----|-----|-----|-----|-----|----|-----|----|-----|-----|-----|-----|
| 0 | 9 | -10 | 11 | -12 | 13 | -14 | 1 | -3 | -9 | 7 | -11 | 6 | -5 | 10 | -4 | 12 | -13 | 14 | -2 | ... |
| 14 | -8 | 0 | 10 | -11 | 12 | -13 | 2 | -1 | 8 | -3 | -10 | 7 | -12 | 13 | -5 | 6 | -4 | 11 | -14 | ... |
| 13 | -14 | 8 | -9 | 0 | 11 | -12 | 3 | -4 | 14 | -2 | 9 | -1 | 7 | -8 | -11 | 5 | -6 | 12 | -13 | ... |
| 12 | -13 | 14 | -8 | 9 | -10 | 0 | 4 | -12 | 13 | -14 | 8 | -3 | 1 | -2 | 10 | -7 | 5 | -9 | 6 | ... |
| -11 | 0 | 13 | -14 | 8 | -9 | 10 | -6 | 11 | 2 | -13 | 14 | -4 | 9 | -1 | 3 | -8 | 7 | -10 | 5 | ... |
| -10 | 11 | -12 | 0 | 14 | -8 | 9 | -5 | 6 | -11 | 12 | 4 | -14 | 3 | -9 | 1 | -2 | 8 | -7 | 10 | ... |
| -9 | 10 | -11 | 12 | -13 | 0 | 8 | -7 | 5 | -10 | 11 | -12 | 13 | -6 | 4 | 2 | -3 | 1 | -8 | 9 | ... |